Evidence for Production of Single Top Quarks at DØ and A First Direct Measurement of $|V_{tb}|$

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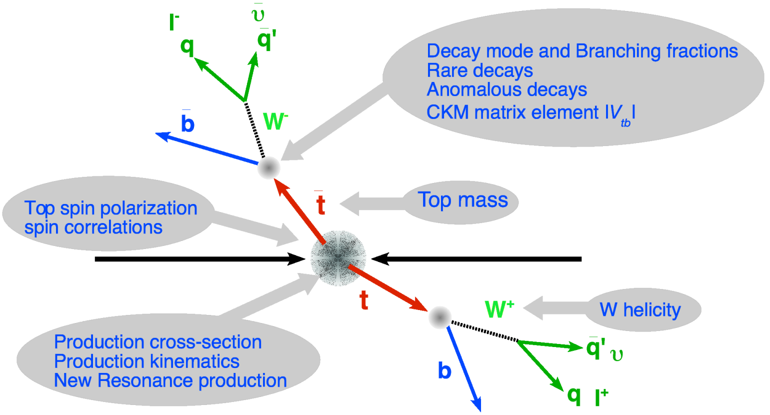
Outline

- Introduction and Motivation
- Preparing for the Measurement
- Multivariate Analysis Techniques
  - Decision Trees
  - Matrix Elements Method
  - Bayesian Neural Networks
- Expected Sensitivity
- Cross Sections and Significance
- First Direct Measurement of $|\mathrm{V_{tb}}|$
- Conclusions

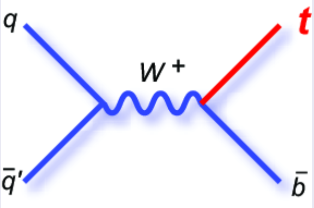
Top Quark Physics

The Tevatron is still the only place to make top quarks.



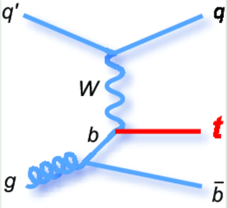
Single top quark production

s-channel (tb)



- $\sigma_{NLO} = 0.88 \pm 0.11$ pb(*)
- current limits (95% C.L.):
  Run II DØ: < 5.0 pb
  (370pb^{-1})
  Run II CDF: < 3.1 pb
  (700pb^{-1})

t-channel (tqb)



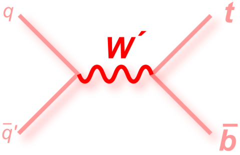
- $\sigma_{NLO} = 1.98 \pm 0.25$ pb (*)
- current limits (95% C.L.):
  Run II DØ: < 4.4 pb
  (370pb^{-1})
  Run II CDF: < 3.2 pb
  (700pb^{-1})

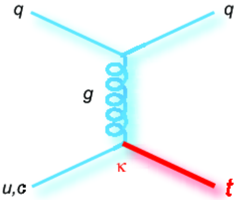
And some very nice CDF results in W&C just last week!!

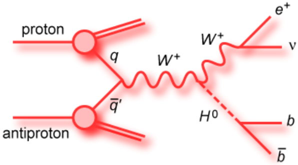
(*) Phys.Rev. D70 (2004) 114012

Motivation

- Directly measure $|\mathrm{V_{tb}}|$ for the first time (more later)
- Cross section sensitivity to beyond the SM processes
- Source of polarized top quarks. Spin correlations measurable in decay products.
- Important background to Higgs search
- Test of techniques to extract a small signal out of a large background





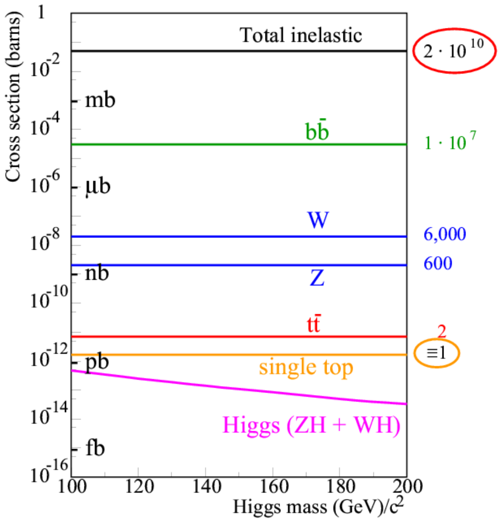


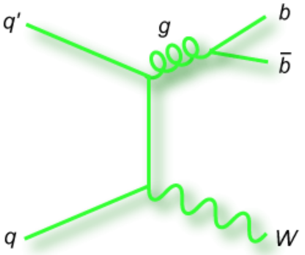
It's not like we haven't been looking already...

1. 2001 Search for electroweak production of single top quarks in ppbar collisions" Phys. Rev. D 63, 031101 (2001)
2. 2001 "Search for Single Top Quark Production at DØ Using Neural Networks," Phys. Lett. **B 517**, 282 (2001).
3. 2004 "Search for Single Top Quark Production at DØ in Run II," DØ Note 4398 (2004).
4. 2005 "Improved Search for Single Top Quark Production," DØ Note 4670 (2005).
5. 2005 "Search for Single Top Quark Production in $p\overline{p}$ Collisions at $\sqrt{s} = 1.96$ TeV," Phys. Lett. **B 622**, 265 (2005).
6. 2006 "Multivariate Searches for Single Top Quark Production with the DØ Detector," submitted to Phys. Rev. D, hep-ex/0604020.

plus 7 PhDs. (CDF has a similar list)

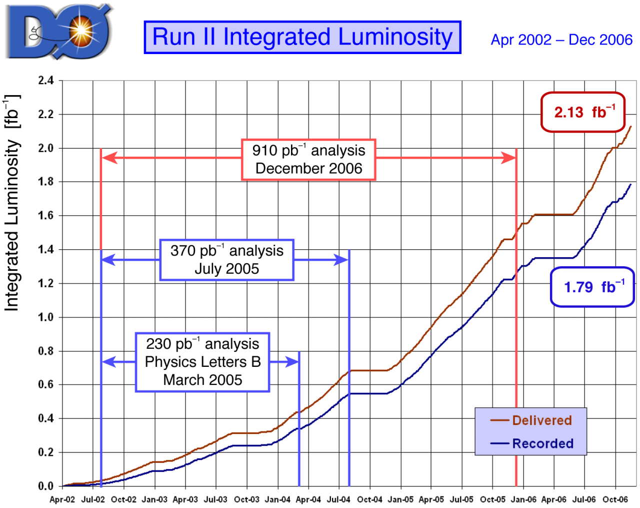
...but it is a challenge!





(stolen from CDF W&C - THANKS!)

Improvements



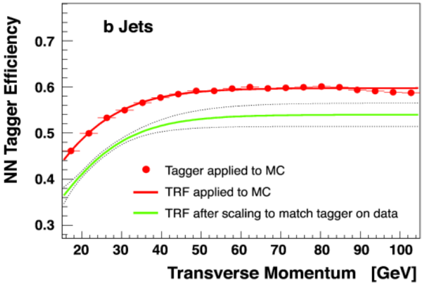
Many thanks to the Accelerator Division

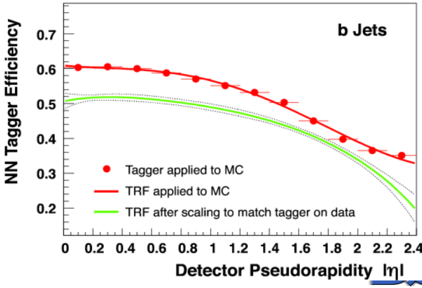
More Improvements...

- Background model improvements
- Fully reprocessed dataset: new calibrations, jet threshold, etc.
- Neural network b-tagging
- Split analysis channels by numbers of jets (exclusive bins)
- Combined $s + t$ search added (SM s:t ratio)

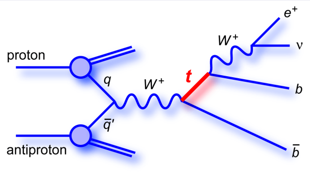
Neural Network b-jet Tagger

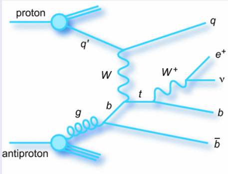
- NN trained on 7 input variables from SVT, JLIP and CSIP taggers.
- Much improved performance!
  - fake rate reduced by 1/3 for same b-efficiency relative to previous tagger
  - smaller systematic uncertainties
- Tag Rate Functions (TRFs) in η, P_T, z-PV applied to MC
- Our operating point:
  - b-jet efficiency $\sim$ 50%
  - c-jet efficiency $\sim$ 10%
  - Light jet efficiency $\sim$ 0.5%





Event Selection



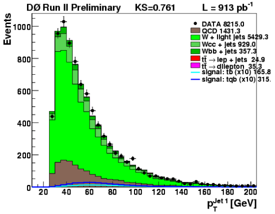


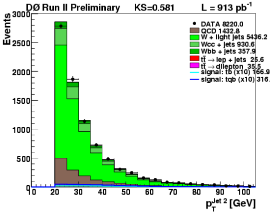
Signature

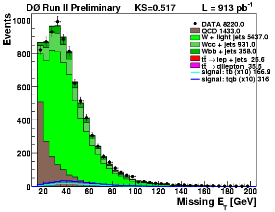
- isolated lepton
- $\not{E}_T$
- 2-4 jets
- at least 1 b-jet

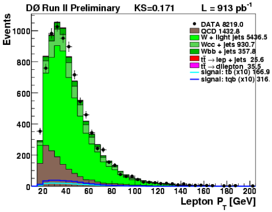
- Only one tight and no other loose lepton
  - electron: $p_T > 15$ GeV and $|\eta_{det}| < 1.1$
  - muon: $p_T > 18$ GeV and $|\eta_{det}| < 2$
- $15 < \not{E}_T < 200$ GeV
- 2-4 jets with $p_T > 15$ GeV and $|\eta_{det}| < 3.4$
  - Leading jet with $p_T > 25$ GeV and $|\eta_{det}| < 2.5$
  - Second leading jet $p_T > 20$ GeV

Event Selection - Agreement Before Tagging









- Normalize W+multijet to data before tagging
- Checked 90 variables,
  3 jet multiplicities,
  1-2 tags,
  electron + muon
- Shown: electron, 2 jets, before tagging
- Good description of data

Event Selection - Yields

Source	Event Yields in 0.9 fb^{-1} Data Electron+muon, 1tag+2tags combined 2 jets	3 jets	4 jets
tb	16 ± 3	8 ± 2	2 ± 1
tqb	20 ± 4	12 ± 3	4 ± 1
$t\bar{t} \to ll$	39 ± 9	32 ± 7	11 ± 3
$t\bar{t} \to l$+jets	20 ± 5	103 ± 25	143 ± 33
W+$b\bar{b}$	261 ± 55	120 ± 24	35 ± 7
W+$c\bar{c}$	151 ± 31	85 ± 17	23 ± 5
W+jj	119 ± 25	43 ± 9	12 ± 2
Multijets	95 ± 19	77 ± 15	29 ± 6
Total background	686 ± 131	460 ± 75	253 ± 42
Data	697	455	246

Event Selection - S/B

Percentage of single top *tb+tqb* selected events and S:B ratio (white squares = no plans to analyze)

Electron + Muon	1 jet	2 jets	3 jets	4 jets	≥ 5 jets
0 tags	10% 1 : 3,200	25% 1 : 390	12% 1 : 300	3% 1 : 270	1% 1 : 230
1 tag	6% 1 : 100	21% 1 : 20	11% 1 : 25	3% 1 : 40	1% 1 : 53
2 tags		3% 1 : 11	2% 1 : 15	1% 1 : 38	0% 1 : 43

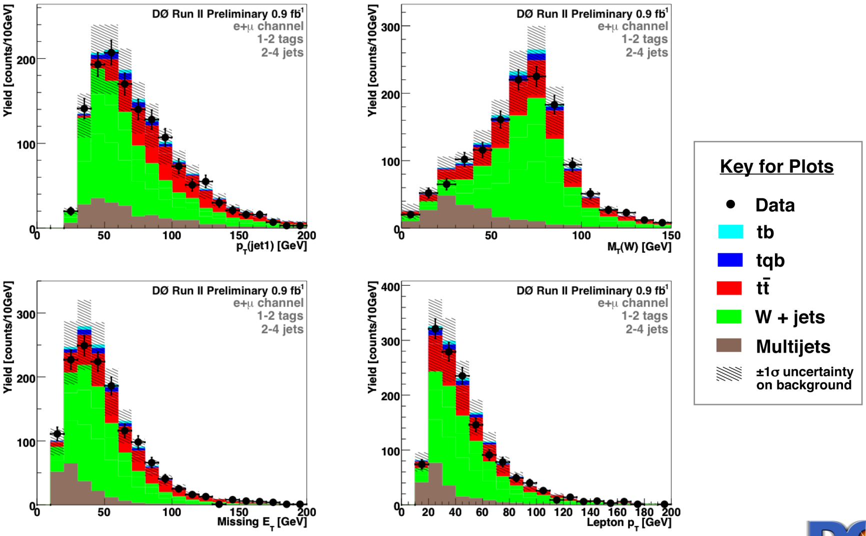
Systematic Uncertainties

- Systematic uncertainties can be either "shaped" (jet energy scale, tag rate functions)
  - Shift inputs by $\pm 1\sigma$, redo analysis
- or "normalization"
  - Uncertainties assigned per background, jet multiplicity, lepton, number of tags

Examples of Relative Systematic Uncertainties

$t\bar{t}$ cross section	18%
Luminosity	6%
Electron trigger	3%
Muon trigger	6%
Jet energy scale	wide range
Jet fragmentation	5–7%
Heavy flavor ratio	30%
Tag-rate functions	2–16%

Systematic Uncertainties

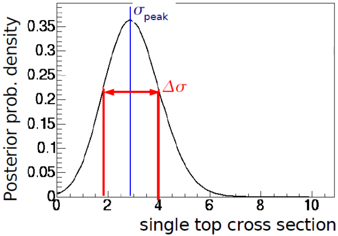


Measuring the Cross Section

Probability to observe data distribution D, expecting y:

$$y = \alpha l \sigma + \sum_{s=1}^{N} b_s \equiv a\sigma + \sum_{s=1}^{N} b_s$$

$$P(D|y) \equiv P(D|\sigma, a, b) = \prod_{i=1}^{nbins} P(D_i|y_i)$$



The cross section is obtained

$$Post(\sigma|D) \equiv P(\sigma|D) \propto \int_a \int_b P(D|\sigma, a, b) Prior(\sigma) Prior(a, b)$$

- Bayesian posterior probability density
- Shape and normalization systematics treated as nuisance parameters
- Correlations between uncertainties properly accounted for
- Flat prior in signal cross section

Ensemble Testing

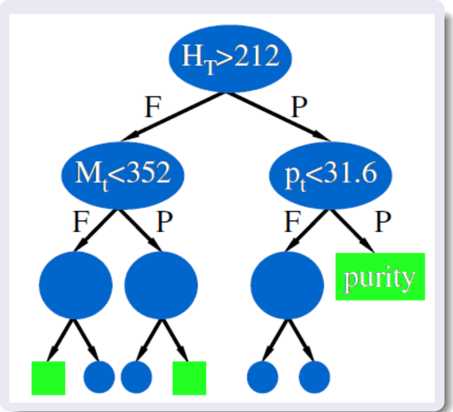
- To verify that all of this machinery is working properly we test with many sets of **pseudo-data**.
- Wonderful tool to test analysis methods! Run DØ experiment 1000s of times!
- Generated ensembles include:
  1. 0-signal ensemble ($s + t\ \sigma = 0pb$)
  2. SM ensemble ($s + t\ \sigma = 2.9pb$)
  3. "Mystery" ensembles to test analyzers ($s + t\ \sigma =??pb$)
  4. Ensembles at measured cross section ($s + t\ \sigma =$measured)
  5. A high luminosity ensemble
- Each analysis tests linearity of "response" to single top.

Multivariate Analysis Techniques

Decision Trees

Train

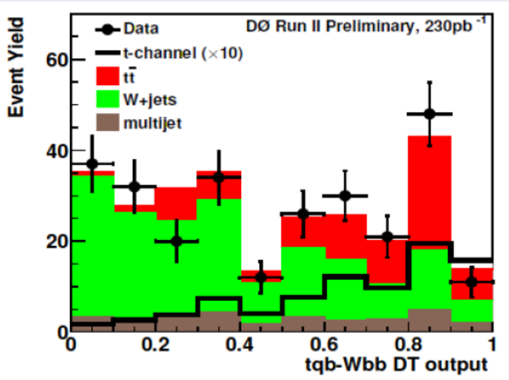
- Start with all events (first node)
- For each variable, find the splitting value with best separation between children (best cut).
- select best variable and cut and produce Failed and Passed branches
- Repeat recursively on each node
- Stop when improvement stops or when too few events left. Terminal node = leaf.



Decision Trees

Measure and Apply

- Take trained tree and run on independent simulated sample, determine purities.
- Apply to Data
- Should see enhanced separation (signal right, background left)
- Could cut on output and measure, or use whole distribution to measure.



Decision Trees - Boosting

Boosting

- Recent technique to improve performance of a weak classifier
- Recently used on DTs by GLAST and MiniBooNE
- Basic principal on DT:
  - train a tree T_k
  - $T_{k+1} = \text{modify}(T_k)$

AdaBoost algorithm

- Adaptive boosting
- Check which events are misclassified by T_k
- Derive tree weight α_k
- Increase weight of misclassified events
- Train again to build T_{k+1}
- Boosted result of event i: $T(i) = \sum_{n=1}^{N_{\text{tree}}} \alpha_k T_k(i)$

- Averaging dilutes piecewise nature of DT
- Usually improves performance

Ref: Freund and Schapire, "Experiments with a new boosting algorithm", in *Machine Learning: Proceedings of the Thirteenth International Conference*, pp 148-156 (1996)

Decision Trees - Application to this Analysis

DT Choices

- 1/3 of MC for training
- Adaboost $\beta = 0.2$
- Boosting cycles $= 20$
- Signal leaf if purity > 0.5
- Minimum leaf size $= 100$ events
- Same total weight to signal and background to start
- Goodness of split - Gini factor

Analysis Strategy

- Train 36 separate trees: (s,t,$s+t$) x (e,μ) x (2,3,4 jets) x (1,2 tags)
- For each signal train against the sum of backgrounds

Decision Trees - 49 variables

Object Kinematics

p_T(jet1)
p_T(jet2)
p_T(jet3)
p_T(jet4)
p_T(best1)
p_T(notbest1)
p_T(notbest2)
p_T(tag1)
p_T(untag1)
p_T(untag2)

Angular Correlations

ΔR(jet1,jet2)
cos(best1,lepton)$_{\mathrm{besttop}}$
cos(best1,notbest1)$_{\mathrm{besttop}}$
cos(tag1,alljets)$_{\mathrm{alljets}}$
cos(tag1,lepton)$_{\mathrm{btaggedtop}}$
cos(jet1,alljets)$_{\mathrm{alljets}}$
cos(jet1,lepton)$_{\mathrm{btaggedtop}}$
cos(jet2,alljets)$_{\mathrm{alljets}}$
cos(jet2,lepton)$_{\mathrm{btaggedtop}}$
cos(lepton,Q(lepton)$\times z$)$_{\mathrm{besttop}}$
cos(lepton,besttopframe)$_{\mathrm{besttopCMframe}}$
cos(lepton,btaggedtopframe)$_{\mathrm{btaggedtopCMframe}}$
cos(notbest,alljets)$_{\mathrm{alljets}}$
cos(notbest,lepton)$_{\mathrm{besttop}}$
cos(untag1,alljets)$_{\mathrm{alljets}}$
cos(untag1,lepton)$_{\mathrm{btaggedtop}}$

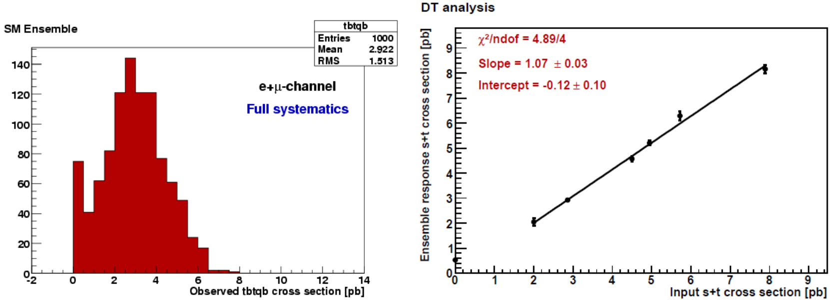
Event Kinematics

Aplanarity(alljets,W)
M(W,best1) ("best" top mass)
M(W,tag1) ("b-tagged" top mass)
H_T(alljets)
H_T(alljets$-$best1)
H_T(alljets$-$tag1)
H_T(alljets,W)
H_T(jet1,jet2)
H_T(jet1,jet2,W)
M(alljets)
M(alljets$-$best1)
M(alljets$-$tag1)
M(jet1,jet2)
M(jet1,jet2,W)
M_T(jet1,jet2)
M_T(W)
Missing E_T
p_T(alljets$-$best1)
p_T(alljets$-$tag1)
p_T(jet1,jet2)
Q(lepton)$\times\eta$(untag1)
$\sqrt{\hat{s}}$
Sphericity(alljets,W)

- Adding variables does not degrade performance
- Tested shorter lists, lose some sensitivity
- Same list used for all channels

Decision Trees - Ensembles

- SM input is returned by DTs
- "Mystery" ensembles are unraveled by the DTs
- Linear response is achieved



Matrix Elements Method - Introduction

A matrix elements analysis takes a very different approach:

- Use the 4-vectors of all reconstructed leptons and jets
- Use matrix elements of main signal and background diagrams to compute an event probability density for signal and background hypotheses.
- Goal: calculate a discriminant:

$$D_s(\vec{x}) = P(S|\vec{x}) = \frac{P_{Signal}(\vec{x})}{P_{Signal}(\vec{x}) + P_{Background}(\vec{x})}$$

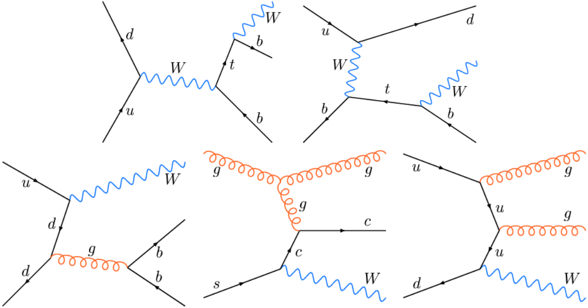
- Define P_{Signal} as properly normalized differential cross section

$$P_{Signal}(\vec{x}) = \frac{1}{\sigma_S} d\sigma_S(\vec{x}) \quad \sigma_S = \int d\sigma_S(\vec{x})$$

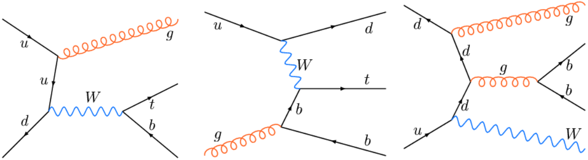
- Shared technology with mass measurement in $t\bar{t}$(eg. transfer functions)

Matrix Elements Method - Introduction

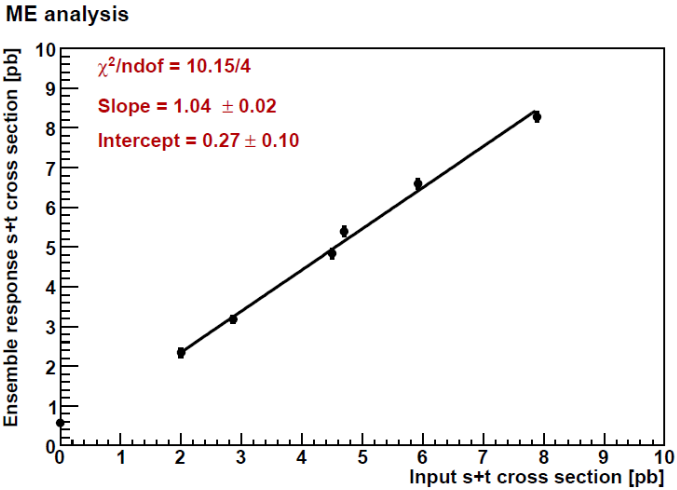
2-jets:



3-jets:



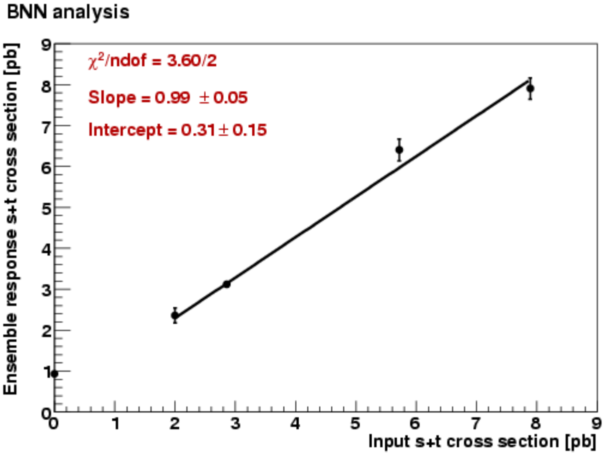
Matrix Elements Method - Ensembles



Bayesian Neural Network - Introduction

- A different sort of neural network:
    - Instead of choosing one set of weights, find posterior probability density over all possible weights
    - Averaging over many networks weighted by the probability of each network given the training data
    - Less prone to overtraining
    - For details see: http://www.cs.toronto.edu/radford/fbm.software.html
- Use 24 variables (subset of DT variables)

Bayesian Neural Network - Ensembles



EXPECTED SENSITIVITY

Significance/Sensitivity Determination

We use our 0-signal ensemble to determine a significance for each measurement.

Expected p-value

The fraction of 0-signal pseudo-datasets in which we measure at least 2.9pb.

Significance/Sensitivity Determination

We use our 0-signal ensemble to determine a significance for each measurement.

Expected p-value

The fraction of 0-signal pseudo-datasets in which we measure at least 2.9pb.

Observed p-value

The fraction of 0-signal pseudo-datasets in which we measure at least the measured cross section.

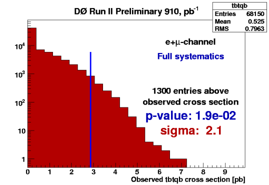
We also can use the SM ensemble to see how compatible our measured value is with the SM.

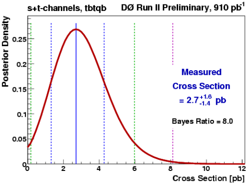
Ensemble Testing - Details

- Use a pool of weighted signal + background events (about 850k in each of electron and muon)
- Fluctuate relative and total yields in proportion to systematic errors
- Randomly sample from a Poisson distribution about the total yield
- Generate a set of pseudo-data (a member of the ensemble)
- Pass the pseudo-data through the full analysis chain (including systematic uncertainties)

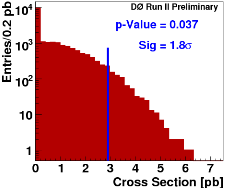
Expected p-value $s + t$

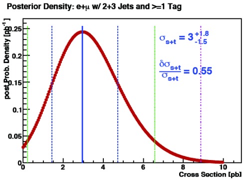
Decision Trees
p-value **1.9%**



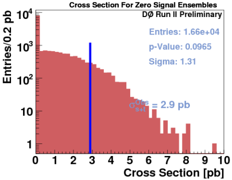


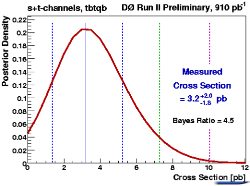
Matrix Elements
p-value **3.7%**





Bayesian NN
p-value **9.7%**

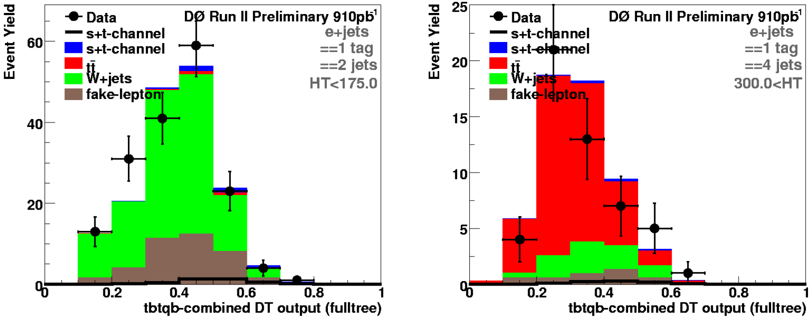




Cross-Checks on Data

Cross-check samples

- "W+jets": $=$2jets, H_T(lepton,$\not\!\!E_T$,alljets) $<$ 175 GeV
- "ttbar": $=$4jets, H_T(lepton,$\not\!\!E_T$,alljets) $>$ 300 GeV
- Shown: $tb+tqb$ DT output for e+jets

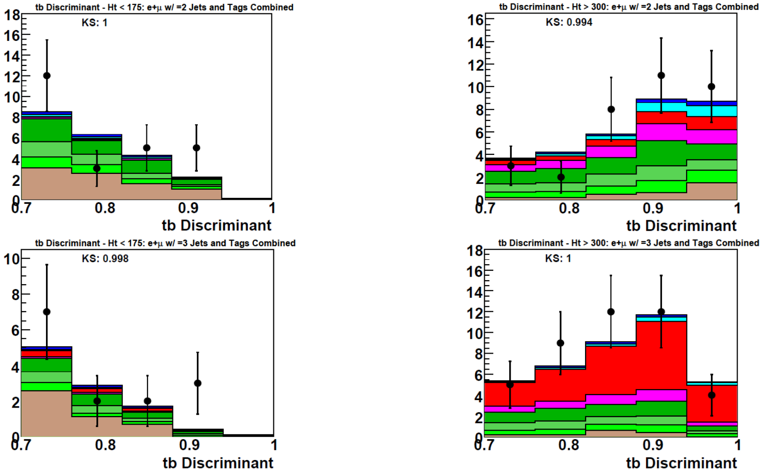


- Good agreement of model with data

Matrix Elements Method - Cross-Checks

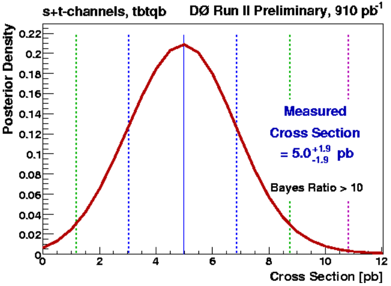
Look at H_T “sidebands” in 2 and 3 jets

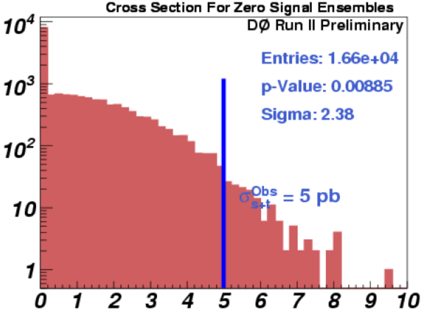
“Soft W-jets” “Hard W-jets”



Cross Sections and Significance

Bayesian Neural Network - Observed

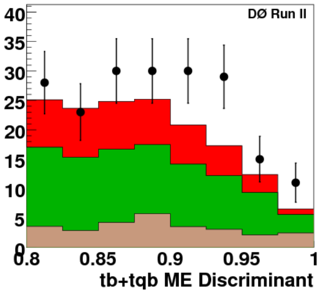


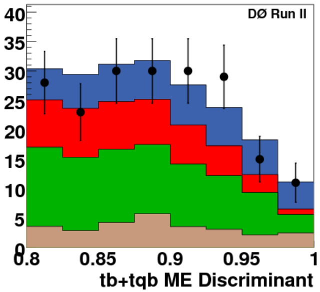


Least sensitive (a-priori) analysis sees 2.4σ effect!

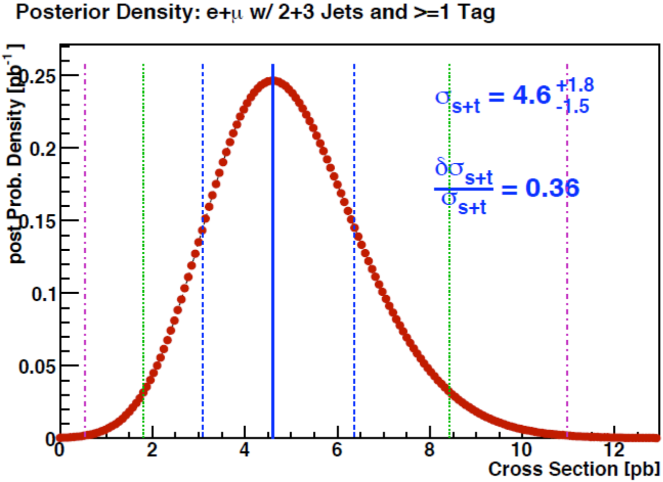
Matrix Elements Method - Observed

Discriminant output with and without signal component (all channels combined in 1D to "visualize" excess)

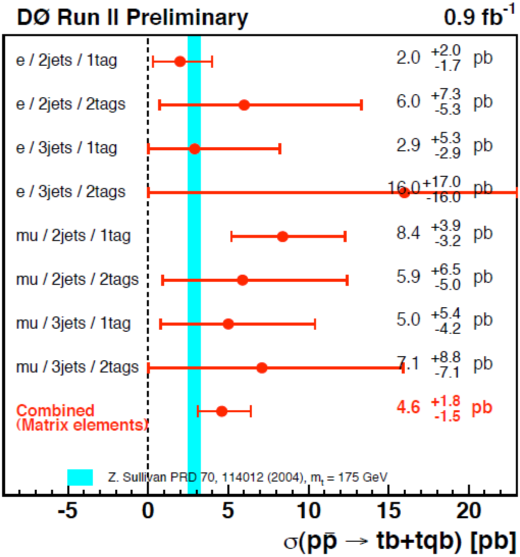




Matrix Elements Method - Observed

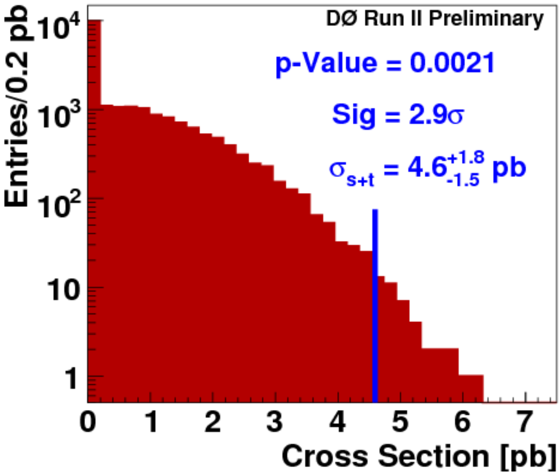


Matrix Elements Method - Summary



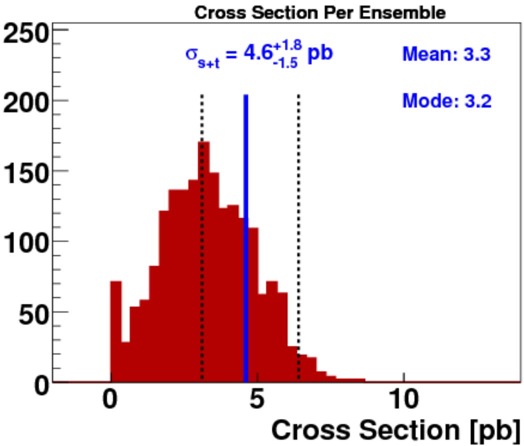
Matrix Elements Method - p-value

p-value$=$0.0021, 2.9σ !!

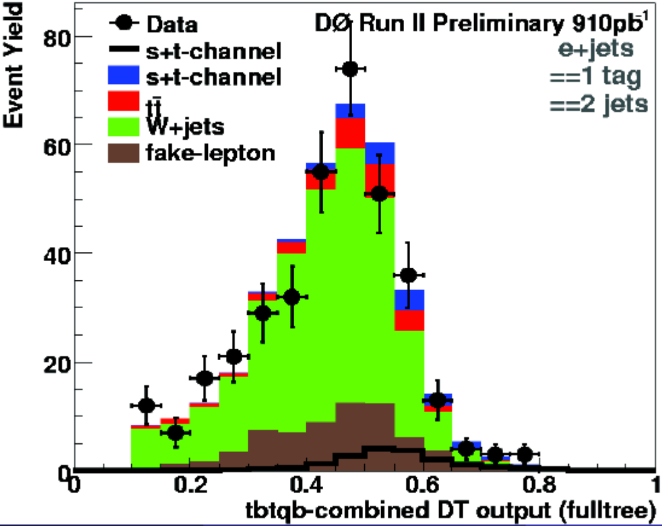


Consistent with SM?

SM compatibility = 21%

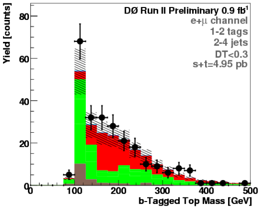


Of course, we have 36 different Decision Trees, let's look at electron, 2 jet, 1 tag:

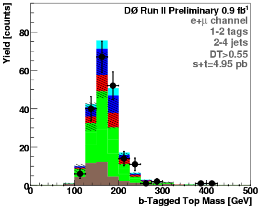


Decision Trees - Event Characteristics $M(W, b)$

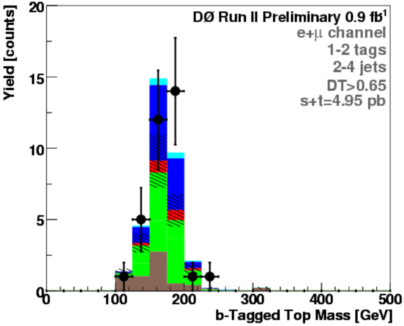
$DT < 0.3$



$DT > 0.55$



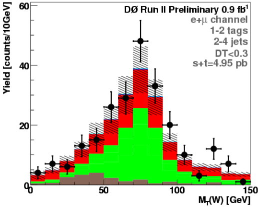
$DT > 0.65$



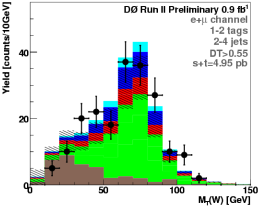
- Excess in high DT output region.

Decision Trees - Event Characteristics M_{Tw}

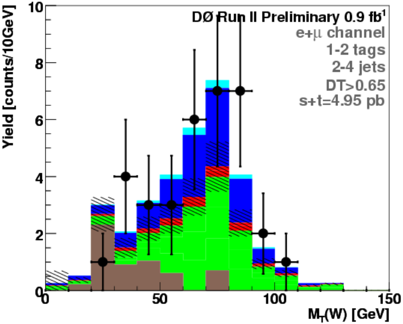
$DT < 0.3$



$DT > 0.55$

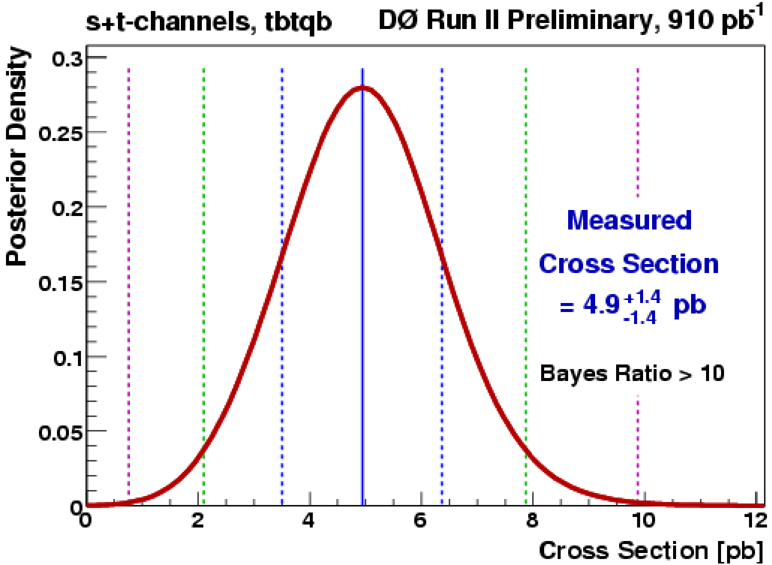


$DT > 0.65$

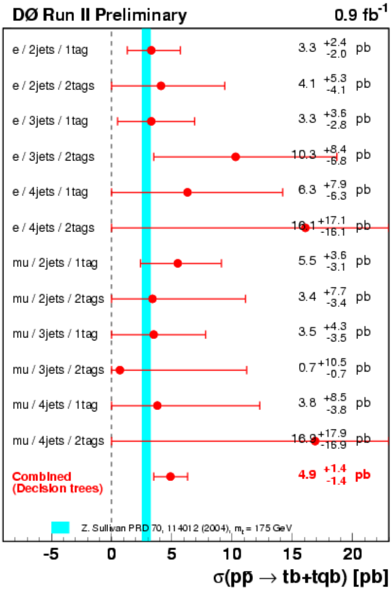


- Excess in high DT output region.

Decision Trees - Observed

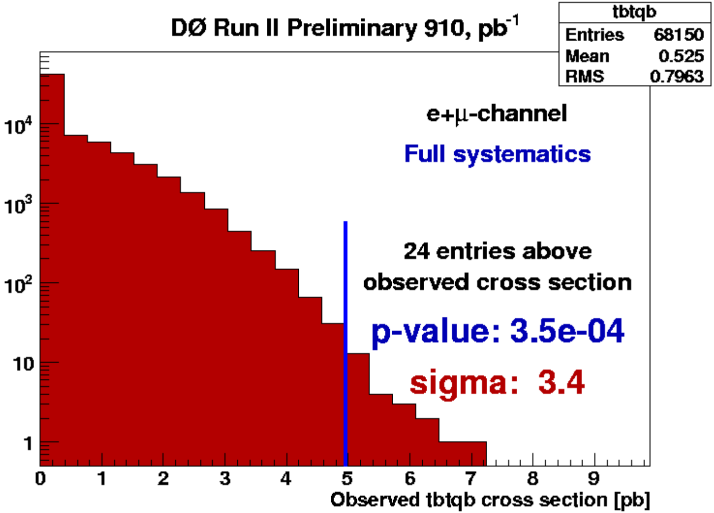


Decision Trees - Summary

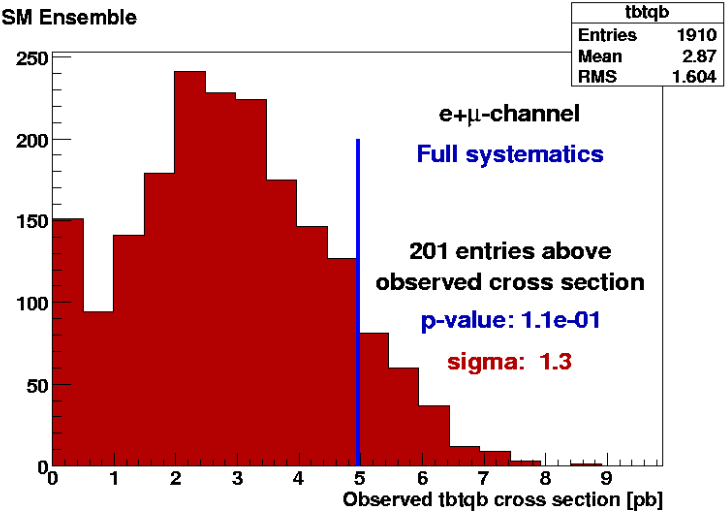


Decision Trees - p-value

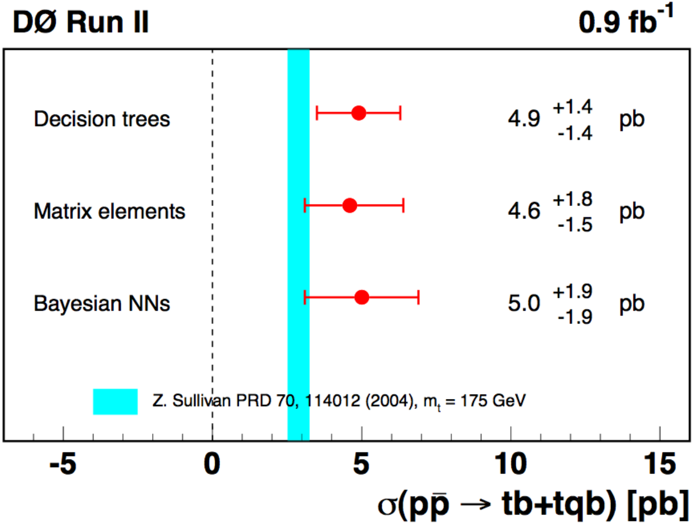
A 3.4σ excess!!



Consistent with SM?



$s + t$ Summary - All methods



Correlations - All methods

Choose the 50 highest events in each discriminant and look for overlap

Technique	Electron	Muon
DT vs ME	52%	58%
DT vs BNN	56%	48%
ME vs BNN	46%	52%

Correlations - All methods

Choose the 50 highest events in each discriminant and look for overlap

Technique	Electron	Muon
DT vs ME	52%	58%
DT vs BNN	56%	48%
ME vs BNN	46%	52%

Also measured the cross section in 400 members of the SM ensemble with all three techniques and calculated the linear correlation between each pair:

	DT	ME	BNN
DT	100%	39%	57%
ME		100%	29%
BNN			100%

Measuring V_{tb}

CKM Matrix Element $\mathrm{V_{tb}}$

Direct access to V_{tb}

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & \color{red}{V_{tb}} \end{pmatrix}$$

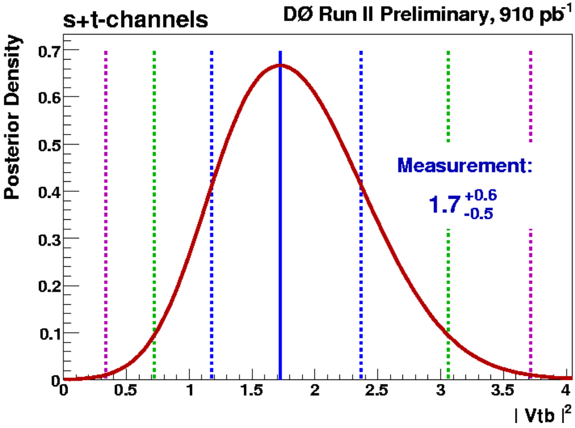
- Weak interaction eigenstates are not mass eigenstates
- In SM: top must decay to a W and d, s or b quark
  - $V_{td}^2 + V_{ts}^2 + V_{tb}^2 = 1$
  - constraints on V_{td} and V_{ts}: $V_{tb} > 0.998$
- New physics that couples to the top quark:
  - $V_{td}^2 + V_{ts}^2 + V_{tb}^2 < 1$
  - no constraint on V_{tb}

Measuring $|\mathrm{V}_{\mathrm{tb}}|$

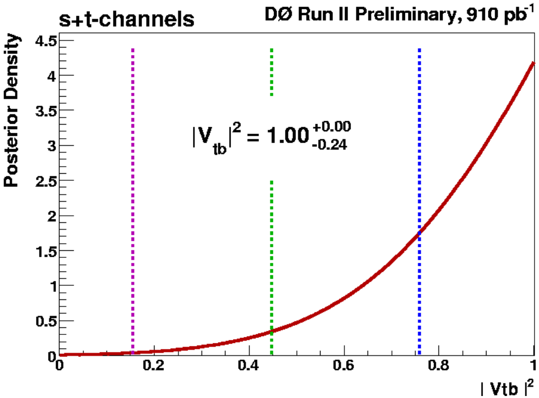
- Given that we now have a measurement of the single top cross section, we can make the first direct meassurement of $|\mathrm{V}_{\mathrm{tb}}|$.
- Use the same infrastructure as cross section measurement but make a posterior in $|\mathrm{V}_{\mathrm{tb}}|^2$.
- Caveat: assume SM top quark decays.
- Additional theoretical errors are needed (see hep-ph/0408049)

	s	t
top mass	13%	8.5%
scale	5.4%	4.0%
PDF	4.3%	10.0%
α_s	1.4%	0.01%

Measuring $|V_{tb}|^2$



Limiting $|V_{tb}|$



Constrain $|V_{tb}|$ to physical region and integrate:

$$|V_{tb}| = 1.00^{-0.12}_{+0}$$

Conclusions

Preliminary First Evidence for Single Top Quark Production!!

- $s + t$ cross section: 4.9 ± 1.4pb
- 3.4σ significance!

- Three techniques in good agreement.
- First direct measurement of $|\mathrm{V_{tb}}|$!!

$$|\mathrm{V_{tb}}| = 1.00^{-0.12}_{+0}$$

BACKUP SLIDES

Electron ID

- We require electrons to be within the central calorimeter: $|\eta^{det}| < 1.1$.
- **Loose isolated electron**
  At least 90% of the energy of the cluster must be contained in the electromagnetic section of the calorimeter. The χ^2 from the 7×7 H-matrix must be less than 50. The energy deposition in the calorimeter must be matched with a charged particle track from the tracking detectors with $p_t > 5$ GeV. Isolation: $(E_{\mathrm{total}}(R < 0.4) - E_{\mathrm{EM}}(R < 0.2))/E_{\mathrm{EM}}(R < 0.2) < 0.15$.
- **Tight isolated electron**
  A tight isolated electron must pass the loose isolation requirements above, and have a value of the seven-variable EM-likelihood $\mathcal{L} > 0.85$.

Muon ID

Loose muons must be of *medium* $|\mathrm{nseg}| = 3$ quality and pass the loose cosmic ray rejection timing requirements: $|\Delta t(\text{A layer scint}, t_0)| < 10$ ns and $|\Delta t(\text{BC layer scints}, t_0)| < 10$ ns. The track reconstructed in the muon system must match a track reconstructed in the central tracker with $\chi^2/\text{ndof} < 4$. The central track is required to have distance of closest approach (dca) to the primary vertex of $|\mathrm{dca}(x, y)| < 0.2$. Note that the previous analysis imposed a dca significance cut of 3 standard deviations that has been removed now. Loose muons must be isolated from jets by $\Delta R > 0.5$.

Tight isolated muon

Tight isolated muons are loose muons with the additional isolation criteria: (a) the momenta of all tracks in a cone of radius $R < 0.5$ around the muon direction, except the track matched to the muon, add up to less than 20% of the muon p_T; and (b) the energy deposited in an annular cone of radius $0.1 < R < 0.4$ around the muon direction is less that 20% of the muon p_T

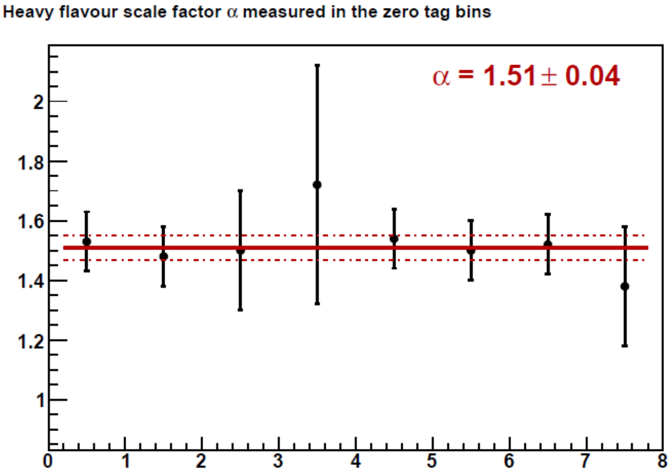
HF Fraction

$$\alpha(Wb\bar{b} + Wc\bar{c}) + Wjj + t\bar{t} + \text{QCD} = \text{Data}$$

Scale Factor α to Match Heavy Flavor Fraction to Data

	1 jet	2 jets	3 jets	4 jets
Electron Channel				
0 tags	1.53 ± 0.10	1.48 ± 0.10	1.50 ± 0.20	1.72 ± 0.40
1 tag	1.29 ± 0.10	1.58 ± 0.10	1.40 ± 0.20	0.69 ± 0.60
2 tags	—	1.71 ± 0.40	2.92 ± 1.20	-2.91 ± 3.50
Muon Channel				
0 tags	1.54 ± 0.10	1.50 ± 0.10	1.52 ± 0.10	1.38 ± 0.20
1 tag	1.11 ± 0.10	1.52 ± 0.10	1.32 ± 0.20	1.86 ± 0.50
2 tags	—	1.40 ± 0.40	2.46 ± 0.90	3.78 ± 2.80

HF Fraction



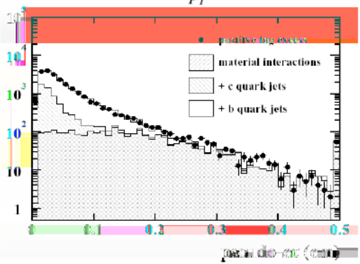
Heavy flavour scale factor α measured in the zero tag bins
α = 1.51 ± 0.04
2
1.8
1.6
1.4
1.2
1
0
1
2
3
4
5
6
7
8

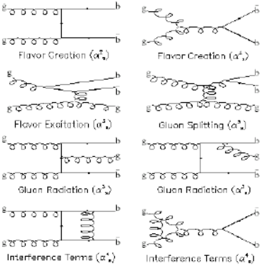
HF Fraction - CDF

1) Estimate generic jet heavy flavor fraction in ALPGEN Monte Carlo
2) Fit for bottom and charm fraction in generic jet data

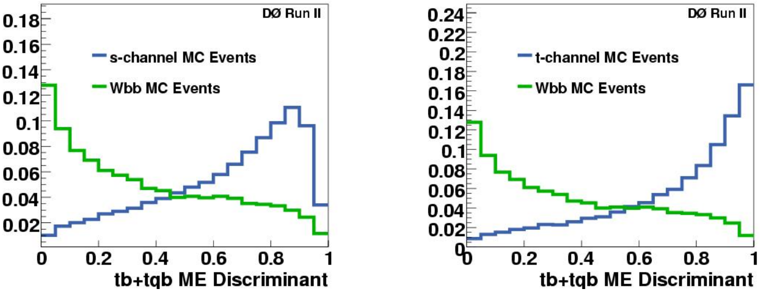
Difference between the two outcomes suggests K=1.5±0.4
Result supported by study using MCFM: J. M. Campbell, J. Houston, Method 2 at NLO, hep-ph/0405276

$$pseudo-c\tau = L_{2D} \cdot \frac{M^{vtx}}{p_T^{vtx}}$$

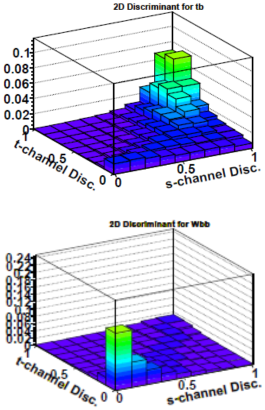


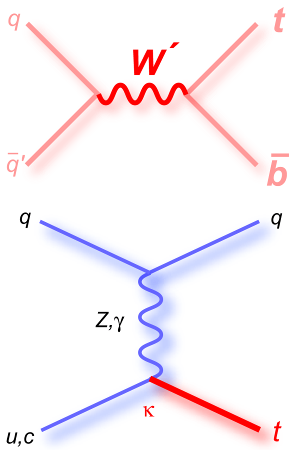


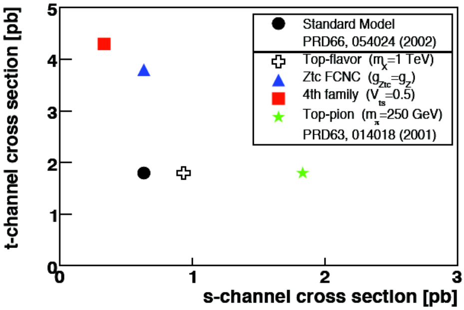
Matrix Element Method



Matrix Element Method




q
t
W´
$\bar{q}'$
$\bar{b}$
q
q
Z,γ
κ
u,c
t


Standard Model
PRD66, 054024 (2002)
Top-flavor (m_x=1 TeV)
Ztc FCNC (g_{Ztc}=g_Z)
4th family (V_{ts}=0.5)
Top-pion (m_π=250 GeV)
PRD63, 014018 (2001)
t-channel cross section [pb]
s-channel cross section [pb]
0
1
2
3
4
5

Uncertainties

Relative Systematic Uncertainties

$t\bar{t}$ cross section	18%	Primary vertex	3%
Luminosity	6%	Electron reco * ID	2%
Electron trigger	3%	Electron trackmatch & likelihood	5%
Muon trigger	6%	Muon reco * ID	7%
Jet energy scale	wide range	Muon trackmatch & isolation	2%
Jet efficiency	2%	$\varepsilon_{\mathrm{real}-e}$	2%
Jet fragmentation	5–7%	$\varepsilon_{\mathrm{real}-\mu}$	2%
Heavy flavor fraction	30%	$\varepsilon_{\mathrm{fake}-e}$	3–40%
Tag-rate functions	2–16%	$\varepsilon_{\mathrm{fake}-\mu}$	2–15%